

# NEW STANDARD ACADEMY

Test Type : Review (Unit Test # 01, 02 & 03)

Do not open this Test Booklet until you are asked to do so. 28-08-2023

## JEE(MAIN): 12<sup>th</sup> Undergoing/Pass Students

Read carefully the Instructions on the Back Cover of this Test Booklet.

### Important Instructions :

1. Immediately fill in the form number on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The candidates should not write their Form Number anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.
3. The Test Booklet consists of 90 questions.
4. There are three parts in the question paper 1,2,3 consisting of Physics, Chemistry and Mathematics having 30 questions in each subject and each subject having Two sections. (i) Section-I contains 20 multiple choice questions with only one correct option. Marking scheme : +4 for correct answer, 0 if not attempted and -1 in all other cases. (ii) Section-II contains 10 Numerical Value Type questions. Attempt any 5 questions. First 5 attempted questions will be considered for marking. Marking scheme : +4 for correct answer, 0 if not attempted and -1 in all other cases.
5. Use Blue/Black Ball Point Pen only for writing particulars/markings responses on Side -1 and Side-2 of the Answer Sheet. Use of pencil is strictly prohibited.
6. No candidate is allowed to carry any textual material, printed or written, bits of papers, mobile phone any electronic device etc, except the Identity Card inside the examination hall/room.
7. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
8. On completion of the test, the candidate must hand over the Answer Sheet to the invigilator on duty in the Room/ Hall. However, the candidate are allowed to take away this Test Booklet with them.

Name of the Candidate(In Capitals) \_\_\_\_\_

Date of Examintation \_\_\_\_\_

Candidate`s Signature: \_\_\_\_\_ Invigilator`s Signature: \_\_\_\_\_

**SECTION-I : (Maximum Marks: 80)**

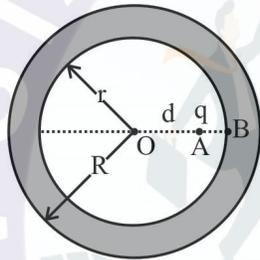
This section contains **20 questions**. Each question has 4 options for correct answer. Multiple-Choice Questions (MCQs) **Only one option is correct**. For each question, marks will be awarded as follows:

*Full Marks* : +4 If correct answer is selected.

*Zero Marks* : 0 If none of the option is selected.

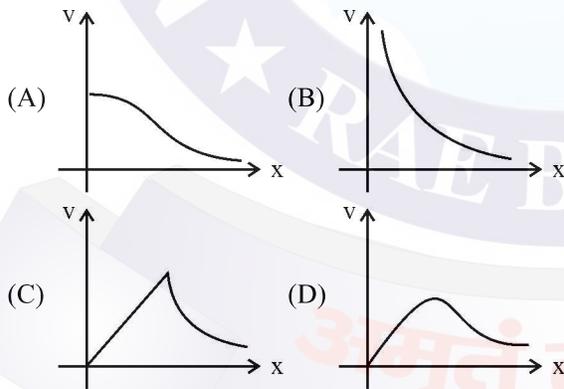
*Negative Marks* : -1 If wrong option is selected.

1. Inside a spherical uncharged conducting shell centered at O, a point charge 'q' is kept at A such that OA = d. The radius of the inner and outer surface of the shell is 'r' and R. The potential of point B is :-

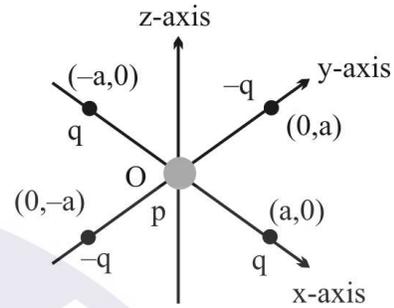


- (A)  $\frac{Kq}{R-d}$  (B)  $\frac{Kq}{R}$  (C)  $\frac{Kq}{r}$  (D)  $\frac{Kq}{d}$

2. The electric potential due to a uniformly charged ring at a point on its axis at a distance x from centre is shown by which of following graphs ?



3. In the figure shown, the equilibrium of proton p is stable with respect to its displacement along



- (A) x-axis (B) y-axis  
(C) z-axis (D) None of these

4. In telescope, if the powers of an objective and eye lens are + 1.25 D and + 20 D respectively, then for relaxed vision, the length of tube and magnification will be :-

- (A) 85 cm and 25 (B) 85 cm and 16  
(C) 21.25 cm and 16 (D) 21.25 cm and 25

5. A telescope has an objective of focal length 100cm and an eyepiece of focal length 2.5 cm. If you look into the objective (that is, into the wrong end) of this telescope, you will see distant objects reduced in size. By what factor will the angular size of objects be reduced in relaxed eye position?

- (A) 40 (B) 10 (C) 20 (D) 4

6. When a telescope is adjusted for parallel light, the distance of the objective from the eyepiece is 100 cm for normal adjustment. The magnifying power of the telescope in this case is 9. If an old man cannot see beyond 90 cm and wishes to use the telescope then he will have to reduce the tube length by

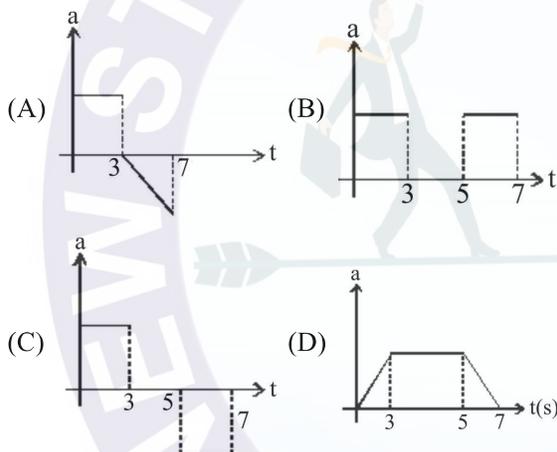
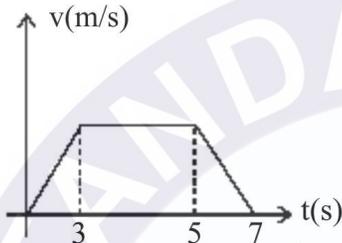
- (A) 1.5 cm (B) 0.5 cm (C) 1 cm (D) 2 cm

Space for Rough Work /

7. A particle is dropped under gravity from rest from a height  $h$  and it travels a distance  $9h/25$  in the last second, the height  $h$  is ( $g = 9.8\text{m/sec}^2$ )

- (A) 100 m (B) 122.5 m  
(C) 145 m (D) 167.5 m

8. For the given velocity time graph of the particle, choose the correct acceleration time graph.



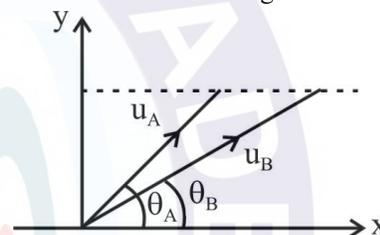
9. A particle moving with speed  $5\text{m/s}$  starts to accelerate. Find the speed at  $t = 3$  sec if the acceleration of the particle varies with time as  $a = t^2 \text{ m/s}^2$ .

- (A)  $32\text{m/s}$   
(B)  $9\text{m/s}$   
(C)  $14\text{m/s}$   
(D)  $4\text{m/s}$

10. Particle is projected with a velocity of  $(10\hat{i} + 12\hat{j}) \text{ m/s}$ . If it have a constant acceleration of  $(3\hat{i} - 3\hat{j}) \text{ m/s}^2$ , then select the correct statement :

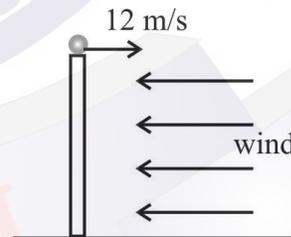
- (A) Initially speed of particle increases  
(B) Initially speed of particle decreases  
(C) Speed remain constant throughout the motion but its direction changes.  
(D) Speed and direction both remain constant.

11. Two particles A and B are projected from the same point in different directions in such a manner that vertical component of their initial velocity are same then ratio of their horizontal range will be equal to :-



- (A)  $\frac{R_A}{R_B} = \frac{u_A}{u_B}$  (B)  $\frac{R_A}{R_B} = \frac{\tan \theta_B}{\tan \theta_A}$   
(C)  $\frac{R_A}{R_B} = \frac{\tan \theta_A}{\tan \theta_B}$  (D)  $\frac{R_A}{R_B} = \frac{\sin \theta_A}{\sin \theta_B}$

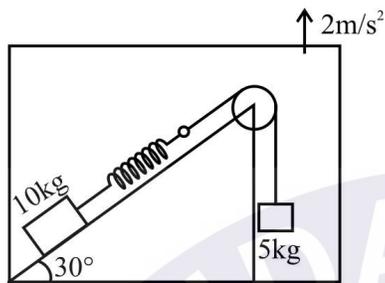
12. A ball is projected horizontally from the top of tower at  $12 \text{ m/s}$  as shown in figure. Wind is blowing as indicated (horizontally), which causes acceleration  $6 \text{ m/s}^2$  horizontally. Displacement of the ball when it strikes the tower, ( $g = 10 \text{ m/s}^2$ )



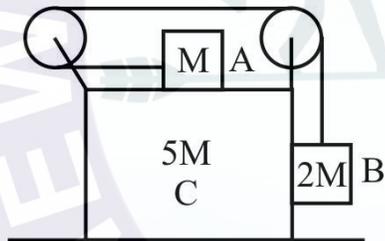
- (A) 20m (B) 60m (C) 80m (D) 40m

Space for Rough Work /

13. In the figure the reading of the spring balance will be (Assume all contact surface is smooth and spring & pulley are ideal) : ( $g = 10\text{m/s}^2$ )

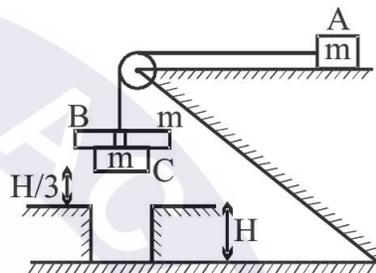


- (A) 50 N  
 (B) 100 N  
 (C) 60 N  
 (D)  $50\sqrt{3}\text{N}$
14. In the system shown the pulleys are ideal and the cords are inextensible (there is no friction anywhere)



- (A) magnitude of acceleration of block A and B will be same  
 (B) magnitude of acceleration of block B and wedge C will be equal  
 (C) magnitude of acceleration of block A will be greater than magnitude of acceleration of wedge C  
 (D) magnitude of acceleration of wedge C will be greater than magnitude of acceleration of block B

15. The system in figure is released from rest from the position shown. After blocks have moved distance  $H/3$ , collar B is removed and block A and C continue to move. What is the speed of C just before it strikes the ground? There is no friction anywhere. Neglect any impulse on string when B is stopped. Neglect size of collar and blocks.



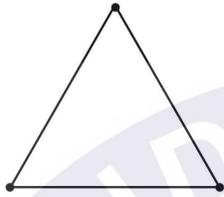
- (A)  $\sqrt{\frac{13gH}{9}}$   
 (B)  $\frac{4}{3}\sqrt{gH}$   
 (C)  $2\sqrt{\frac{gH}{3}}$   
 (D)  $\frac{\sqrt{10gH}}{3}$

16. A body attains a height equal to the radius of the Earth when projected from Earth' surface. The velocity of the body with which it was projected is :-

- (A)  $\sqrt{\frac{GM_e}{R}}$   
 (B)  $\sqrt{\frac{2GM_e}{R}}$   
 (C)  $\sqrt{\frac{5}{4} \frac{GM_e}{R}}$   
 (D)  $\sqrt{\frac{3GM_e}{R}}$

Space for Rough Work /

17. If the three point masses are released simultaneously to move under influence of mutual gravitational attraction, speed of each particle when each one of them has moved a distance  $\left(\frac{a}{2\sqrt{3}}\right)$ , is :-

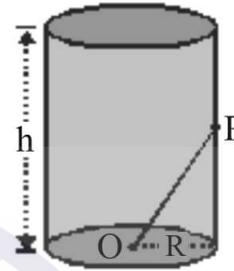


- (A)  $\sqrt{\frac{6\sqrt{3} Gm}{(2-\sqrt{3}) a}}$   
 (B)  $\sqrt{\frac{2 Gm}{a}}$   
 (C)  $\sqrt{\frac{Gm}{a}}$   
 (D) None of these

18. A particle is fired vertically from the surface of earth with a velocity  $kv_e$ , where  $v_e$  is the escape velocity and  $k < 1$ . Neglecting air resistances, the height to which it will rise from surface of earth ( $R =$  radius of earth) is

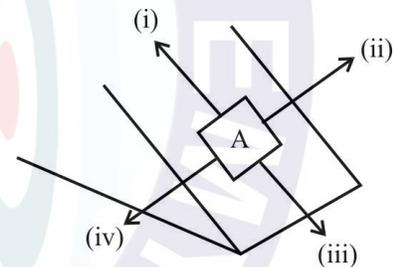
- (A)  $\left(\frac{k}{1-k}\right) R$   
 (B)  $\left(\frac{k+1}{k-1}\right) R$   
 (C)  $\left(\frac{k^2}{1-k}\right) R$   
 (D)  $\left(\frac{k^2}{1-k^2}\right) R$

19. Electric field at point P is given by  $\vec{E} = E_0 \vec{r}$ . The total flux through the given cylinder of radius R and height h is:-



- (A)  $E_0 \pi R^2 h$   
 (B)  $2E_0 \pi R^2 h$   
 (C)  $3E_0 \pi R^2 h$   
 (D)  $4E_0 \pi R^2 h$

20. A block stays in equilibrium on an inclined plane. It is easiest to push in the direction shown as :-



- (A) (i)  
 (B) (ii)  
 (C) (iii)  
 (D) (iv)

Space for Rough Work /

**SECTION-II : (Maximum Marks: 20)**

**This section contains 10 questions Candidates have to attempt any 5 questions out of 10. If more than 5 questions are attempted, then only first 5 attempted questions will be evaluated.**

The answer to each question is a **Numerical Value**.

For each question, enter the correct integer value (In case of non-integer value, the answer should be rounded off to the nearest Integer).

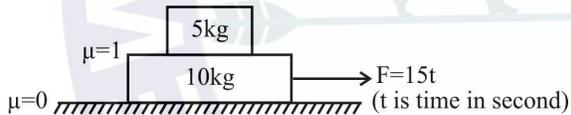
Answer to each question will be evaluated according to the following marking scheme:

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*Negative Marks* : -1 If wrong answer is entered.

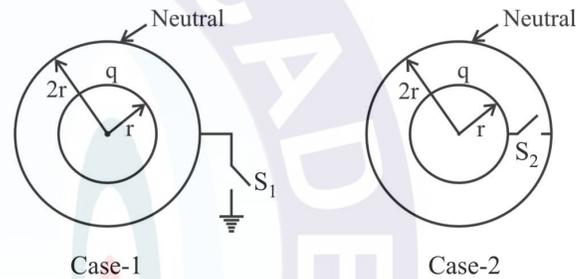
1. In the figure shown, surface is smooth whereas, coefficient of friction between 10 kg and 5 kg block is  $\mu = 1$ . A force  $F = 15t$  acts on 10 kg in horizontal direction. Find time (in seconds) by which work done by friction on 5 kg block is 10 joule.



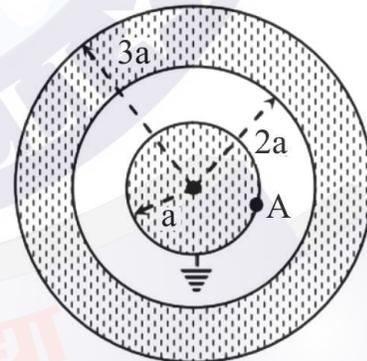
2. A compound microscope is used to enlarge an object kept at a distance of 3 cm from its objective. The objective consists of several convex lenses in contact and has a focal length of 2 cm. If a lens of focal length 10 cm is removed from the objective, the eyepiece has to be moved by  $x$  cm to refocus the image. The value of  $x$  is
3. A body starts from rest and moves for 'n' seconds with uniform acceleration 'a', its velocity after n seconds is 8 m/s. The displacement of the body in last 3 seconds is 15 m. Find a (in  $\text{m/s}^2$ ).

4. Let the maximum horizontal range a particle can achieve with an initial speed  $u$  is  $R$  for ground to ground projection. If a particle is projected with speed  $u$ , has a horizontal range  $\frac{3R}{5}$  then difference in the maximum heights attained in the two cases is  $\frac{R}{n}$ , then find the value of  $n$ .

5. In given two cases two concentric conducting spherical shells are arranged as shown. In case-1 when switch  $S_1$  is closed, energy loss is  $Q_1$ . In case-2 when switch  $S_2$  is closed, energy loss is  $Q_2$  then find  $Q_1 - Q_2$

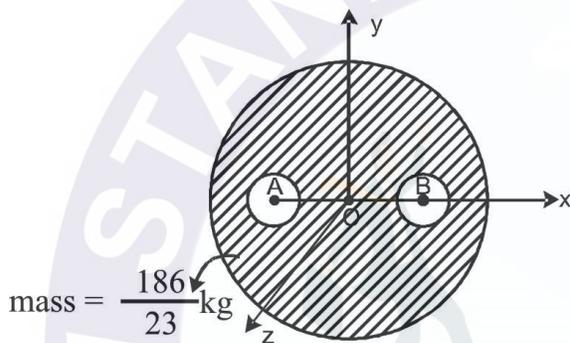


6. A conducting sphere of radius  $a$  is inside a hollow conducting sphere of radii  $2a$  and  $3a$  as shown in the figure. The inner sphere is earthed. The capacitance of system of sphere and hollow sphere is  $5\pi k \epsilon_0 a$ . Find the value of  $k$ .

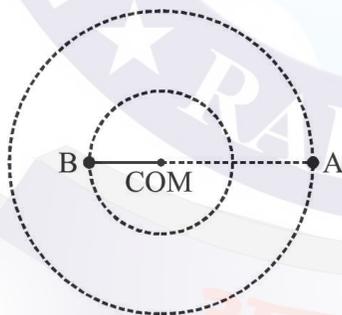


Space for Rough Work /

7. A solid sphere of uniform density and radius  $4m$  is located with its centre at the origin  $O$  of coordinate system. Two spheres of equal radii  $1m$  each, with their centres at  $A(-2m, 0, 0)$  and  $B(2m, 0, 0)$  are taken out of the solid sphere leaving behind spherical cavities as shown in the figure and remaining body has mass of  $\frac{186}{23} \text{ kg}$ . Gravitational potential at the centre of the sphere is given by  $V_0 = -GX \text{ joule/kg}$ . Find the value of 'X'.

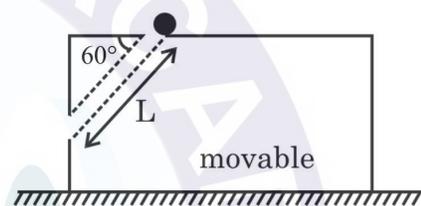


8. Figure shows a binary star system revolving about their COM. The masses of star A & B are  $15 \times 10^{30} \text{ kg}$  and  $45 \times 10^{30} \text{ kg}$  respectively. Find the ratio of area swept by star A to area swept by star B about their common COM in a common time interval.



9. Electric field due to a uniformly charged ring of radius  $\sqrt{5} \text{ m}$  at a certain point on its axis is  $1 \text{ V/m}$  and electric potential at the same point is  $6 \text{ V}$ . If the distance of this point from centre of the ring is  $x$  (in m), find the smallest value of  $x$ .

10. Shown in the figure is a block having an inclined smooth groove in vertical plane. A ball of same mass is released at rest from top end. Time when ball will leave groove is  $\sqrt{\frac{nL}{2g\sqrt{3}}}$ . Find  $n$ .



Space for Rough Work /

## PART-2 : CHEMISTRY

### SECTION-I : (Maximum Marks: 80)

This section contains **20 questions**. Each question has 4 options for correct answer. Multiple-Choice Questions (MCQs) **Only one option is correct**. For each question, marks will be awarded as follows:

**Full Marks** : +4 If correct answer is selected.

**Zero Marks** : 0 If none of the option is selected.

**Negative Marks** : -1 If wrong option is selected.

1. A 2 L sample of a gaseous hydrocarbon is burnt in excess oxygen. The only products of the reaction are 8L of  $\text{CO}_2(\text{g})$  and 10L of  $\text{H}_2\text{O}(\text{g})$ , all at  $100^\circ\text{C}$  and 1 atm pressure. The molecular formula of the hydrocarbon is -

(A)  $\text{C}_5\text{H}_{12}$  (B)  $\text{C}_4\text{H}_5$  (C)  $\text{C}_4\text{H}_{10}$  (D)  $\text{C}_8\text{H}_{10}$

2. To find formula of compound composed of A & B which is given by  $\text{A}_x\text{B}_y$ , it is strongly heated in oxygen as per reaction-



If 2.5 gm of  $\text{A}_x\text{B}_y$  on oxidation gives 3 gm oxide of A, Find empirical formula of  $\text{A}_x\text{B}_y$ , [Take atomic mass of A = 24 & B = 14]

(A)  $\text{A}_3\text{B}_2$  (B)  $\text{A}_2\text{B}_3$  (C)  $\text{AB}_2$  (D)  $\text{A}_2\text{B}$

3. Antifluorite structure is -

(A) 4 : 4 coordination of anion to cation

(B) 4 : 8 coordination of anion to cation

(C) 8 : 4 coordination of anion to cation

(D) 6 : 6 coordination of anion to cation

4. Due to Frenkel defect, the density of the ionic solids :

(A) Increases (B) Decreases

(C) Does not change (D) Changes

5. Which one of following pairs of aqueous solution will not be isotonic at same temperature

(A) 1M NaCl and 2 M urea

(B) 1.5 M  $\text{AlCl}_3$  and 2M  $\text{Na}_2\text{SO}_4$

(C) 2.5 M KCl and 1M  $\text{Al}_2(\text{SO}_4)_3$

(D) 1M  $\text{CaCl}_2$  and 1M NaCl

6. 2.0 g of benzoic acid dissolved in 25.0g of benzene shows a depression in freezing point equal to 1.96K. Molal depression constant ( $K_f$ ) of benzene is  $4.9 \text{ K.kg.mol}^{-1}$ . What is the percentage association of the acid?

(A) 78 % (B) 72 % (C) 82 % (D) 68 %

7. A catalyst lowers the activation energy of a reaction from  $20 \text{ kJ mole}^{-1}$  to  $10 \text{ kJ mole}^{-1}$ . The temperature at which the uncatalysed reaction will have the same rate as that of the catalysed at  $27^\circ\text{C}$  is:-

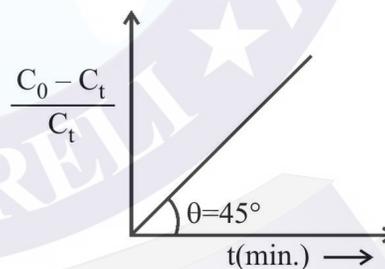
(A)  $-123^\circ\text{C}$

(B)  $327^\circ\text{C}$

(C)  $150^\circ\text{C}$

(D)  $+23^\circ\text{C}$

8. For II order reaction  $\text{A} \rightarrow \text{P}$ , graph is



Then  $t_{1/2}$  of reaction is ?

(A) 10 min.

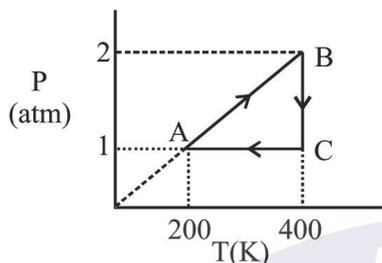
(B) 15 min.

(C) 60 sec.

(D) 120 sec.

Space for Rough Work /

9. One mole of an ideal monoatomic gas undergoes following reversible cyclic process -



Magnitude of total work done in a complete cycle in litre - atm is

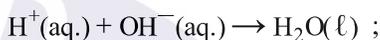
(Take :  $R = 0.08 \text{ L-atm/mole-K}$ ,  $\ln 2 = 0.7$ )

- (A) Zero  
 (B) 6.4 litre-atm  
 (C) 200 litre-atm  
 (D) 16 litre-atm
10. 1.6 gm  $\text{CH}_4(\text{g})$  undergo combustion at constant volume at 300K and 2.5 kcal of heat is released. Enthalpy of combustion ( $\Delta_{\text{C}}\text{H}$ ) of  $\text{CH}_4$  will be:
- (A) 27 kcal/mol (B) - 26.2 kcal/mol  
 (C) +23 kcal/mol (D) - 4.5 kcal/mol
11. The  $\Delta G$  in the process of melting of ice at  $-10^\circ\text{C}$  and 1 bar pressure is -
- (A)  $\Delta G < 0$  (B)  $\Delta G > 0$   
 (C)  $\Delta G = 0$  (D)  $\Delta G \leq 0$
12. The entropy change when 2 moles of an ideal monoatomic gas is subjected to change in state from (1atm, 10l) to (2 atm, 5l) will be : [ $\ln 2 = 0.7$ ]
- (A) - 2.8 cal/K (B) 2.8 J/K  
 (C) - 1.4 cal/K (D) 5.6 J/K

13.  $\text{P}_4\text{O}_6(\text{g}) \rightarrow 4\text{P}(\text{g}) + 6\text{O}(\text{g})$   $\Delta H = +x \text{ kJ/mole}$  then  $\text{BE}_{\text{P-O}}$  will be -

- (A)  $\frac{x}{6} \text{ kJ}$  (B)  $\frac{x}{12} \text{ kJ}$   
 (C)  $83.33 \times x \text{ kJ}$  (D)  $x \text{ kJ}$

14. 0.2 M, 100 ml NaOH is mixed with 0.4 M, 100 ml HCl solution. Determine energy released during reaction -



$$\Delta H = -57.5 \text{ kJ/mol}$$

- (A) 1150 J (B) 1150 kJ  
 (C) 2300 J (D) 2300 kJ

15. In which of the following species, lone pair occupying orbital of central atom has maximum percentage of s-character ?

- (A)  $\text{:}\ddot{\text{C}}\text{IF}_3$  (B)  $\ddot{\text{N}}\text{H}_3$   
 (C)  $\ddot{\text{P}}\text{H}_3$  (D)  $\text{H}_2\ddot{\text{O}}$

16. Which of the following order is correct for covalent character (if the cation sizes are identical in pairs)-

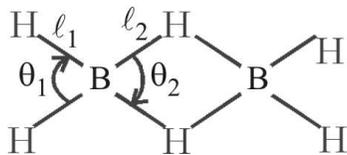
- (A)  $\text{NaCl} < \text{CuCl}$   
 (B)  $\text{CaCl}_2 < \text{PdCl}_2$   
 (C)  $\text{KCl} < \text{AgCl}$   
 (D) All of these

17. Which of the following oxyacid consists of X-O-X linkage. (X = S, P or N)

- (A)  $\text{H}_2\text{S}_2\text{O}_5$  (B)  $\text{H}_4\text{P}_2\text{O}_5$   
 (C)  $\text{H}_2\text{S}_2\text{O}_3$  (D)  $\text{H}_2\text{N}_2\text{O}_2$

Space for Rough Work /

18. Observe the following conversion and structure of  $B_2H_6$  (Diborane)  $2BH_3(g) \longrightarrow B_2H_6(g)$



The correct statement is :

- (A)  $l_1 > l_2$   
 (B)  $\theta_1 > \theta_2$   
 (C)  $B_2H_6$  consists 3c-2e bond and is a planar species  
 (D) The hybridisation state of Boron atom in  $B_2H_6$  is not changed as compared to monomeric form of  $BH_3$
19. Correct trend of first ionisation energy in group-13 is
- (A)  $B > Al > Ga > In > Tl$   
 (B)  $B > Al > Ga > Tl > In$   
 (C)  $B > Tl > Ga > Al > In$   
 (D)  $B > Ga > Al > In > Tl$
20. Calculate the density (in gm/ml) of aqueous NaOH solution of which molarity & (%w/w) are equal

- (A) 8  
 (B) 4  
 (C) 2  
 (D) 1

### SECTION-II : (Maximum Marks: 20)

This section contains 10 questions Candidates have to attempt any 5 questions out of 10. If more than 5 questions are attempted, then only first 5 attempted questions will be evaluated.

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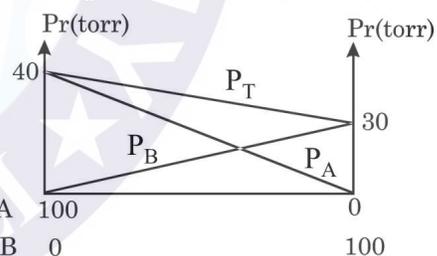
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Negative Marks : -1 If wrong answer is entered.

- The half lives of decomposition of gaseous  $CH_3CHO$  at constant temperature but at initial pressure of 364 mm and 170 mm Hg were 410 second and 880 second respectively. Hence order of reaction is
- Equal mass of A & B are present in liquid solution then total pressure exerted by vapours is X torr, Give X/6. [A = 80 gm/mol, B = 120 gm/mol]

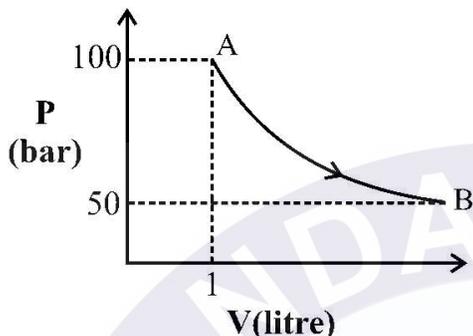


- Ar crystallizes in fcc arrangement and density of solid and liquid Ar 3.7 and 3 gm/cc respectively. Find percentage of empty space in liquid Ar.

(Write your answer as sum of digit)

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4. Find the heat absorbed by an ideal gas (in kJ) when it follows the graph (given below) during an isothermal expansion [ $\ln 2 = 0.7$ ] [Take : 1 litre-bar = 0.1 kJ]



5. 100 ml of 0.3M HCl is mixed with 200 ml of 0.3 M  $\text{H}_2\text{SO}_4$ . Calculate the final molarity of the  $\text{H}^+$  ions in the resulting solution.

**Fill your answer by multiplying with 10**

6. The mass of 'O' in 1 mole  $\text{KClO}_3$  is x gm. Then  $\frac{x}{8}$  is :
7. Maximum number of degenerate orbitals in M-shell of  $\text{Li}^{+2}$  ?
8. Find the number of ions having ionic radius greater than  $\text{Na}^+$ .  
 $\text{K}^+, \text{Rb}^+, \text{Mg}^{+2}, \text{F}^-, \text{O}^{2-}, \text{Al}^{+3}$
9. Find the number of B–O–B linkage in the anionic part of borax.
10. Calculate the magnitude of work done by 2mole ideal gas in kJ, when expansion is taking place from 2 litre to 22 litre against constant external pressure of 1 atm.

[Given : 1 litre-atm = 100 J]

### PART-3 : MATHEMATICS

#### SECTION-I : (Maximum Marks: 80)

This section contains **20 questions**. Each question has 4 options for correct answer. Multiple-Choice Questions (MCQs) **Only one option is correct**. For each question, marks will be awarded as follows:

*Full Marks* : +4 If correct answer is selected.

*Zero Marks* : 0 If none of the option is selected.

*Negative Marks* : -1 If wrong option is selected.

1. The value of  $\sum_{r=181}^{269} \log_{10}(\tan r)$  is
- (A) a natural number  
(B) a negative integer  
(C) a positive real number  
(D) a rational number
2. If  $f(x) = x^{11} + \sin^3(35x) + 111x$ , then the value of  $f^{-1}\left(\sin\frac{\pi}{5}\right) + f^{-1}\left(\sin\frac{6\pi}{5}\right) + f^{-1}\left(\sin\frac{\pi}{7}\right) + f^{-1}\left(\sin\frac{8\pi}{7}\right)$  is equal to -
- (A)  $f(\pi^{11})$  (B)  $f\left(\frac{\pi}{7}\right)^{11}$   
(C)  $f\left(\frac{\pi}{5}\right)^{11}$  (D)  $f(0)$
3. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  &  
 $f(x) = \frac{\sin([x]\pi)}{x^2 + 2x + 3} + 2x - 1 + \sqrt{x(x-1) + \frac{1}{4}}$   
(where  $[x]$  denotes integral part of  $x$ ), then  $f(x)$  is -
- (A) one-one but not onto  
(B) one-one & onto  
(C) onto but not one-one  
(D) neither one-one nor onto

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4. The ratio of the greatest value of  $2 - \cos x + \sin^2 x$  to its least value is  
 (A)  $\frac{7}{4}$  (B)  $\frac{11}{4}$  (C)  $\frac{13}{4}$  (D)  $\frac{9}{4}$
5. If  $y = (\sin \theta + \cos \theta)^2 + (\cos \theta + \sec \theta)^2$ , then minimum value of  $y$  is :  
 (A) 2 (B) 5 (C) 8 (D) 9
6. Let  $f(x)$  be a polynomial of degree 6 divisible by  $x^3$ , and having a point of extremum at  $x = 2$ . If  $f'(x)$  is divisible by  $1 + x^2$ , then find the value of  $\frac{3f(2)}{f(1)}$ .  
 (A) 14 (B) 15  
 (C) 16 (D) None of these
7. The slope of the tangent to the curve represented by  $x = t^2 + 3t - 8$  and  $y = 2t^2 - 2t - 5$  at the point  $M(2, -1)$  is  $\left(\frac{\lambda}{7}\right)$ , then value of  $\lambda$  is ?  
 (A) 4 (B) 5 (C) 6 (D) 7
8. Water is leaking at the rate of  $2m^3/\text{sec}$  from bottom of an inverted cone of semi-vertical angle  $45^\circ$ . If the rate at which periphery of water surface changes when the height of the water in the cone is 2 metres is  $d$  m/sec, then  $|5d|$  is equal to :  
 (A) 4 (B) 5 (C) 6 (D) 7
9. The real numbers  $a, b, c$  and  $d$  are each less than or equal to 12. The polynomial  $P(x) = ax^3 + bx^2 + cx + d$  ( $a \neq 0$ ) satisfies  $P(2) = 2$ ,  $P(4) = 4$  and  $P(6) = 6$ , then the maximum possible value of  $P(10)$  is -  
 (A) 10 (B) 42  
 (C) 58 (D) 106
10. For the curve  $by^2 = (x+a)^3$  the square of subtangent is proportional to  
 (A) (Subnormal) $^{1/2}$  (B) Subnormal  
 (C) (Subnormal) $^{3/2}$  (D) None of these
11. Let  $C$  be the curve  $y = x^3$  (where  $x$  takes all real values). The tangent at  $A$  meets the curve again at  $B$ . If the gradient at  $B$  is  $K$  times the gradient at  $A$  then  $K$  is equal to  
 (A) 4 (B) 2 (C)  $-2$  (D)  $\frac{1}{4}$
12. If  $f$  is twice differentiable such that  $f''(x) = -f(x)$  and  $f(x) = g(x)$ . If  $h(x)$  is a twice differentiable function such that  $h'(x) = (f(x))^2 + (g(x))^2$ . If  $h(0) = 2$ ,  $h(1) = 4$  then the equation  $y = h(x)$  represents:  
 (A) a curve of degree 2  
 (B) a curve passing through the origin  
 (C) a straight line with slope 2  
 (D) a straight line with  $y$  intercept equal to 1
13. If the equation  $x^4 - (12K + 5)x^2 + 16K^2 = 0$  ( $K > 0$ ) has four real solutions which are in A.P. then the value of  $K = \frac{a}{b}$  where  $a$  and  $b$  are co-prime, then find the value  $(a - 3b)$ ?  
 (A) 3 (B) 4  
 (C) 6 (D) 7
14. If  $\sum_{n=1}^{2015} (-1)^n \left(\frac{n^2 + n + 1}{(n)!}\right) = -a - \frac{b}{c!}$  (where  $a, b, c \in \mathbb{N}$ ), then the minimum value of  $\left(\frac{a + b + c}{576}\right)$  is  
 (A) 5 (B) 7 (C) 9 (D) 3

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15. If  $x, y, z$  are non-zero real numbers, then the minimum value of the expression

$$\frac{(x^8 + x^4 + 1)(y^8 - y^4 + 1) \left( z^8 + \frac{1}{3}z^4 + 1 \right)}{x^4 y^4 z^4} \text{ is}$$

- (A) 5 (B) 4 (C) 7 (D) 2

16. If sum of first  $n$  terms of an A.P. is  $\forall n \in \mathbb{N}$ , then the value of

$$\sum_{n=1}^{\infty} \frac{21}{(S_n S_{n+2} + S_{n-1} S_{n+1}) - (S_n S_{n+1} + S_{n-1} S_{n+2})}$$

- (A) 3 (B) 2 (C) 1 (D) 8

17. Let  $f(x) = (1+x) \ln(1+x) - x - \frac{x^2}{4}, \forall x > -1$  and  $f(x) > 0 \forall x \in (0, \alpha)$ , then value of  $[\alpha]$  is (where  $[\cdot]$  denotes greatest integer function)

- (A) 11 (B) 6 (C) 2 (D) 4

18. Number of integral values of  $p$  for which the cubic equation  $2x^3 - 3x^2 + p = 0$  has 3 real roots, is

- (A) 1 (B) 2 (C) 3 (D) 4

19. Complete set of values of  $m$ , for which function  $f(x) = 3 + mx + e^{-x}$  is always decreasing, is -

- (A)  $[0, \infty)$  (B)  $(-\infty, 0]$   
(C)  $[2, 5]$  (D)  $[7, 17]$

20.  $f(x) = x(1-x)^3$  then which of the following is true

- (A)  $f(x)$  has local maxima at  $x = 1$   
(B)  $f(x)$  has local minima at  $x = 1$   
(C)  $f(x)$  has local minima at  $x = \frac{1}{4}$   
(D)  $f(x)$  has local maxima at  $x = \frac{1}{4}$

## SECTION-II : (Maximum Marks: 20)

This section contains 10 questions Candidates have to attempt any 5 questions out of 10. If more than 5 questions are attempted, then only first 5 attempted questions will be evaluated.

The answer to each question is a Numerical Value.

For each question, enter the correct integer value (In case of non-integer value, the answer should be rounded off to the nearest Integer).

Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 If correct answer is entered.

Zero Marks : 0 If the question is unanswered.

Negative Marks : -1 If wrong answer is entered.

1. Let ABC be a triangle right angled at C. The value of arithmetic mean of  $\frac{1}{\log_{c-b} a}$  and  $\frac{1}{\log_{c+b} a}$  is (where  $a, b, c$  denotes length of sides BC, AC, AB respectively and  $b+c \neq 1, c-b \neq 1$  and  $a > 1$ )

2. Let  $f(x) = \begin{cases} \cos^{-1} \left( \frac{1+x}{\sqrt{2(1+x^2)}} \right) ; & x \leq 0 \\ \tan^{-1} x ; & x > 0 \end{cases}$ . If the range of value of  $k$  for which the equation  $f(x) = k$  has exactly two solutions is  $[m, n)$  then find the value of  $\left( \frac{\pi}{2} \left( \frac{1}{m} + \frac{1}{n} \right) + 2 \right)$ ?

3. Number of integral roots of the equation

$$8x^{100} - 20x^4 + 3x^2 - 91 = 0$$

4. If the roots of the equation  $ax^2 + bx + c = 0$ , are of the form  $\frac{\alpha}{\alpha-1}$  &  $\frac{\alpha+1}{\alpha}$  then the value of  $\frac{(a+b+c)^2 + 4ac}{b^2}$  is equal to

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5. Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined as  $f(x) = x + \cos x + 2$  and  $g(x)$  be the inverse function of  $f(x)$ . Find  $(g'(3) + g''(3))$

6. If for a continuous function  $f$ ,  $f(0) = f(1) = 0$  and  $f'(1) = 2$  and  $g(x) = f(e^x) \cdot e^{f(x)}$ , then  $g'(0)$  is equal to-

7. If  $f(x) = \frac{1}{x^2 - 17x + 66}$ , then  $f\left(\frac{2}{x-2}\right)$  has a removable discontinuity at how many points?

8. Sum of Integral values of 'x' satisfying

$$\sqrt{(\log_2 x + \log_x(16x) - 5)(\log_2 x) +}$$

$$\sqrt{(\log_2 x + \log_x(2x) - 3)(\log_2 x)} = 1 \text{ is}$$

9. Find the number of integral value(s) in the domain of  $f(x) = \sqrt{\ln|\ln|x||} + \sqrt{7|x| - |x|^2} - 10$ ?

10. If a function  $f(x)$  is defined by

$$f(x) = x + \ln\left(\frac{x}{1-x}\right) \text{ and let the value of the sum}$$

$$f\left(\frac{1}{209}\right) + f\left(\frac{2}{209}\right) + f\left(\frac{3}{209}\right) + \dots + f\left(\frac{208}{209}\right)$$

is  $N$ , then  $\frac{N}{13}$  is equal to

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